Plenary Lecture 4

Mathematical Work of Teaching in Multilingual Context

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ABSTRACT It is well acknowledged that teaching mathematics in multilingual classrooms where the language of teaching and learning is not the students' home language presents challenges. It is also well acknowledged that there are various knowledge demands on teaching mathematics, and that teachers face different tasks that constitute mathematical work of teaching. In this paper I explore the link between the two by conceptualising the mathematical work of teaching in multilingual contexts. I do this by drawing on lessons from studies on mathematical knowledge for teaching and mathematical work of teaching, from studies on teaching mathematics in multilingual contexts, and from my work in Malawi. My conceptualisation yields four categories, and I illustrate these using some examples from the context of Malawi.

Keywords Multilingual contexts; mathematical work of teaching; Malawi.

1. Introduction

In this paper I discuss the mathematical work of teaching in multilingual contexts. "Mathematical work of teaching" are the tasks that teachers are regularly faced with as they teach mathematics (Ball, Thames, and Phelps, 2008). In the discussion I make reference to mathematical knowledge for teaching, and by this I use the definition by Ball et al. (2008, p. 395) to mean the "the mathematical knowledge needed to carry out the work of teaching mathematics". I will use the terms 'multilingual classroom' and 'multilingual context' to refer to all classrooms and contexts that have one or more languages besides the language of teaching and learning. While I acknowledge the differences between bilingual and multilingual that have been highlighted by many researchers, the differences are not crucial for this paper.

Discussing mathematical work of teaching in multilingual context is complex because it involves two large fields of research in mathematics education: (i) teaching mathematics in multilingual contexts and (ii) mathematical work of teaching and mathematical knowledge for teaching. Language and the teaching and learning of mathematics has been researched in different parts of the world. The emphasis and focus of the research has varied depending on the contexts and issues relevant to the situations at hand. Across the various contexts and research studies, the issues appear

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to be similar in general ways but also different in specific ways. For example, one general similarity is that the various studies uncover some challenges in teaching and learning mathematics that come about because of language, while the specific nature of the challenges vary across the contexts.

When we look at mathematical work of teaching and Mathematical Knowledge for Teaching (MKT), research studies in this field generally agree that teaching mathematics requires different types of knowledge in addition to the subject matter knowledge. There are also some variations in this field in the way researchers conceptualise and name the various types of teacher knowledge. In comparison to the research on language and mathematics, in the research on MKT, there seem to be more similarities than differences. This could be because the focus is on mathematics and the mathematical knowledge demands on teaching, which can be explored in a similar manner across various contexts despite the different languages or language contexts. However, the demands on teaching might not be similar.

My work in mathematics education over the years has been mainly in these two fields and my research has mostly been in Malawi, my home country, and occasionally in other parts of south and east Africa through collaborations with other researchers. My experience, therefore, is that of multilingual classrooms where the language of teaching and learning is English, but not the home language of the students and the teachers. Malawi uses English as the language of teaching and learning from the fifth year of primary school onwards. In the first four years of primary school, local languages are used, however, textbooks for these first four years are in Chichewa only, the national language. Chichewa is also taught as a subject from the first year and throughout primary and secondary schools. Therefore the case of Malawi is often bilingual for students and teachers that have Chichewa as their home language, or trilingual (Phakeng, Planas, Bose, and Njurai, 2018) for those that have other home languages. While the classrooms as a whole can have multiple languages, thus multilingual.

My recent work in Malawi in collaboration with colleagues from University of Stavanger aims at improving the quality of mathematics education in schools through professional development of teachers and teacher educators. What has been evident in all my work and through my post graduate students' studies is that language is always an issue in such contexts even if we attempt to focus on different areas of research and not foreground language. The same is likely to be the case for others in the different multilingual contexts. Therefore it is important to discuss the work of teaching mathematics in multilingual contexts.

Researchers that have studies mathematical work of teaching include Ball and her colleagues at University of Michigan, and Adler and her colleagues at University of the Witwatersrand. Studying the mathematical work of teaching is important because "strengthening our understanding of the mathematical work of teaching … is a critical dimension of enhancing its teaching and learning" (Adler, 2010: p. 122). Ball et al (2008) suggested 16 tasks that teachers are faced with when teaching mathematics, and Adler (2010) elaborated 4 tasks of teaching mathematics. All these tasks describe the

mathematical work of teaching that is universal and apply to various contexts around the world. While these universal tasks strengthen our understanding of the mathematical work of teaching, they do not describe fully the work of teaching in specific contexts such as multilingual classrooms. It is therefore important to study the specific contexts and also strengthen our understanding of the work of teaching in such contexts.

I build from previous research to conceptualise the mathematical work of teaching in multilingual contexts. The main question I am exploring is: what is the mathematical work of teaching in multilingual contexts? I answer this question through the following three guiding questions and I relate to the context of Malawi and my work over the years.

(1) What lessons can we draw from studies on teaching and learning mathematics in multilingual contexts?

(2) What lessons can we draw from studies on mathematical work of teaching and mathematical knowledge for teaching?

(3) How can these inform conceptualisation of mathematical work of teaching in multilingual context?

My discussion is presented in four sections. First, I discuss the lessons we learn from studies on teaching in multilingual classrooms. Second, I discuss lessons from studies on mathematical knowledge for teaching. Third, I relate the discussion to the context of Malawi, the research studies in Malawi and lessons we learn. Finally, I discuss how the lessons can inform conceptualisation of mathematical tasks of teaching in multilingual classrooms.

2. Lessons from studies on language and mathematics teaching

Research on language and mathematics teaching can be grouped in a number of categories. I consider the three main categories as (a) those that focus on the mathematical language and terminology, which concerns all students even first language speakers of the language of instruction; (b) those that focus on students from minority language groups such as immigrant families into a country or community where the language of instruction is not their home language, for example Spanish language immigrants in the United States; (c) those that focus on students in countries where the language of instruction is a foreign language and not the students home language, for example previously colonised countries that use the colonial language. The third category is the case for many African countries.

I acknowledge the work by many other scholars who have focused on language and communication in mathematics and those that have discussed theoretical resources that can be used to study and understand use of language in mathematics (e.g. Sfard, 2008; Barton, 2008; Barwell, 2007). Morgan, Craig, Schutte and Wagner (2014) highlight the importance of theorising when studying language and communication in mathematics education. They point out that while there are many studies that focus on

language use and how it contributes to students' learning of specific mathematics concepts, "little attention is given to the more general issue of the acquisition of mathematical ways of speaking or writing that may be applicable and acceptable in a wide range of areas of mathematics" (p. 852). Morgan et al. (2014) identify three areas of concern in this regard, one of which is "what knowledge and skills might teachers need to use in order to support the development of students' linguistic mathematical competence". This is an important concern which is relevant to this paper, and relates to questions raised by other researchers discussed later in the paper.

2.1. Lessons on mathematical language and terminology

As early as the work of Pimm (1987) and Orton (1992), we learn that some mathematical vocabulary pose problems for students, even for first language speakers. For example, words that have everyday meanings that are different from the mathematical meanings. This is difficult for all students to learn the precise mathematical meanings and use appropriately. Research and studies in this category inform us of the need to understand the complexity of mathematical vocabulary, to pay attention to words that might be difficult for students, and to plan carefully how to teach the meanings and use of such words in mathematics lessons.

2.2. Lessons from studies on classrooms with minority language groups

Examples of this work are that of Moschkovich in the US, Planas in Catalonia, and Barwell in the UK and Canada. The lessons we learn from these include that students from minority languages who are not fluent in the language of instruction need special attention to make the mathematics accessible to them. Without deliberate attempts to include these students in mathematics lessons, they will not access the mathematics and thus we will not achieve equity in mathematics education. Planas (e.g. Planas, 2019; Planas and Setati-Phakeng, 2014; Planas and Civil, 2013) and Moschkovich (e.g. Moschkovich, 2012; 2018) also caution against perceiving learners as deficit, and they suggest instead to view the learners' home languages as resources.

2.3. Lessons from studies on classrooms with foreign language as the language of teaching and learning

The context of teaching and learning in a foreign language that is not the students' home language is common in most of Sub-Sahara Africa. The foreign language is usually the colonial language, which after independence has continued to be used as official and school language. For example, English in Malawi, French in Cameroon and Portuguese in Mozambique. My review of research in this category is limited to English as a foreign language. Most of the research in sub-Sahara Africa on teaching and learning in multilingual contexts has been done in South Africa (e.g. Adler, 2001; Setati and Adler, 2000; Setati, 2008; Setati, Molefe, and Langa, 2008; Essien, 2010; Webb and Webb, 2008). There are at least seven general and related lessons that we learn from the studies:

(1) Students and parents prefer use of English as the language of teaching and learning because English is the language of power and brings "social goods" (Setati, 2008).

(2) Use of English or home language in teaching and learning should not be taken as a dichotomy where use of one completely excludes use of the other (Setati, 2008).

(3) Students' home languages should be seen as resources and not problems (Setati, 2008).

(4) Code switching can be used effectively where teachers and students share a common language (Webb and Webb, 2008).

(5) Code switching presents teachers with challenges and dilemmas (Setati and Adler, 2000; Adler, 2001; Setati, 2001)

(6) Bilingual approach where use of home language alongside English is planned and used proactively can be an effective way of teaching mathematics (Setati et al., 2008)

(7) Teacher education does not prepare teachers adequately for teaching in multilingual classrooms (Chitera, 2009; Essien, 2010)

(8) Setati, Chitera and Essien (2009) reviewed research of mathematics education in multilingual classrooms in South Africa, and raised three important questions, one of which is: "What do all teachers need to know, and what skills do they need to develop in order to be able to teach mathematics effectively in multilingual classrooms?" (p. 76). This is an important question that I will address in this paper, but with a focus on Malawi context.

3. Lessons from studies on mathematical knowledge for teaching

Since Shulman's seminal papers (Shulman, 1986, 1987) on teacher knowledge and his conceptualisation of subject matter knowledge (SMK) and pedagogical content knowledge (PCK), many researchers have based their studies on Shulman's work and elaborated on the categories of knowledge for teaching mathematics (e.g. Ball et al., 2008; Rowland, Huckstep, and Thwaites, 2005). I will focus on the work of Ball et al. (2008) that suggest six domains of teacher knowledge — three under SMK and three under PCK, which they illustrate in a figure shown in Fig. 1.

One of the lessons we learn from this work is that there are different types of knowledge that are needed for teaching mathematics. We also learn from their further work that these can be measured through carefully designed items. Review of MKT research and publications (Hoover, Mosvold, Ball, and Lai, 2016; p.9) reveals that although there have been many studies on the nature and composition of knowledge for teaching mathematics, it is difficult to draw lessons because most of these studies "do not build on each other in obvious ways and clear lessons are hard to identify". Hoover et al. (2016; p. 9) acknowledge that "the one avenue of work that represents progress for the field is the development of instruments." According to the review, there are many studies that have teacher education as a priority; these include studies

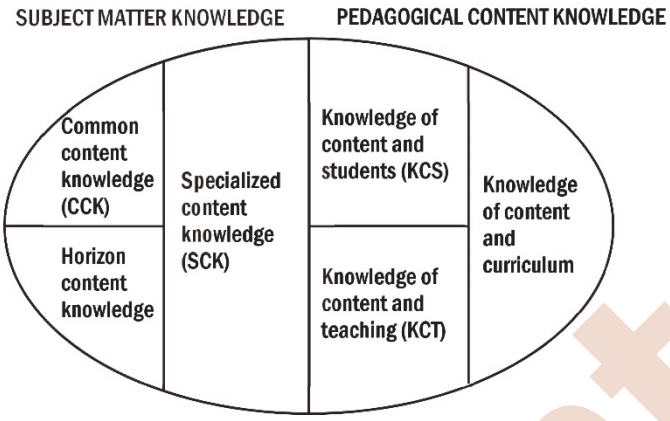

Fig. 1 Domains of MKT (Ball et al., 2008, p. 403)

on the design and evaluation of teacher education programmes. Hoover et al. (2016; p.11–12) suggest the following as what they observe as related emerging lessons from the several decades of research:

- Teaching teachers additional standard disciplinary mathematics beyond a basic threshold does not increase their knowledge in ways that impact teaching and learning.
- Providing teachers with opportunities to learn mathematics that is intertwined with teaching increases their mathematical knowledge for teaching.
- The focus of the content, tasks, and pedagogy for teaching such knowledge requires thoughtful attention to ways of maintaining a coordination of content and teaching without slipping exclusively into one domain or the other.

These are important lessons, because the main goal of studying MKT is that we understand it in ways that inform our work with teachers – both pre-service and in-service. However, I add that each of these requires careful consideration depending on the context, school curriculum and other issues surrounding teaching of mathematics in the specific context. For example, the basic threshold might vary across different school curricula, and identifying these might not be easy. Similarly designing and planning mathematics that is intertwined with teaching depends on the context: the curriculum, resources, language, and other characteristics of the context.

Hoover, Mosvold and Fauskanger (2014) shift the focus from knowledge for teaching to tasks of teaching, pointing out that there seem to be no discussion among the researchers of mathematical knowledge for teaching, about identifying common tasks of teaching that would form an international body of knowledge. They suggest that "the idea of common tasks of teaching – that represent a decomposition of the work of teaching into professionally recognizable components – constitutes a potential foundation for an internationally useful practice-based theory of MKT" (p. 8). Hoover et al. (2014; 7) call for "increased efforts to identify professionally defensible

mathematical tasks of teaching that can serve as a common foundation for conceptualizing and measuring mathematical knowledge for teaching internationally"

What Hoover et al. (2014) are calling for is similar to what Adler (2010) did earlier when she elaborated on some tasks of teaching mathematics drawn from studies of mathematics classrooms in South Africa. She identified 'designing, adapting and selecting tasks', 'processes and objects', 'valuing and evaluating diverse learner productions' as interrelated mathematical tasks of teaching. These seem to be examples of "common tasks of teaching" Hoover et al. (2014) are calling for. As Adler explains, the identified tasks of teaching illustrate some of the mathematical knowledge for teaching discussed by Ball et al. (2008), thus building onto the body of knowledge.

Ball (2017; 29) also shifts focus from knowledge domains to specific mathematical work of teaching and argues that "the quest to answer the perennial question of what mathematical 'knowledge' teachers need should be based on a deep and nuanced understanding of what teachers actually do".

I respond to this shift in focus and conceptualise the mathematical work of teaching in multilingual contexts and not knowledge domains. I do this by drawing on the professional knowledge that I extract from the lessons learnt from the previous studies.

3.1. MKT studies in multilingual contexts

Studies of MKT in Africa are not many (Jakobsen and Mosvold, 2015). The available studies use the theories from University of Michigan; the earlier ones (e.g. Adler, 2005; Kazima and Adler, 2006; Kazima, Pillay and Adler, 2008) use Ball, Bass and Hill (2004) aspects of mathematical problem solving that teachers do, while the later ones (e.g. Adler, 2010; Mwadzaangati, 2018; Mamba, 2018) use Ball et al. (2008) domains of MKT. Lessons from these studies in African multilingual contexts include that the tasks of teaching as identified in the US are also identified in these contexts, furthermore there are some additions to the work of teaching that include paying attention to language; for example what is said and how it is said (Kazima and Adler, 2006). This suggests that teachers in this context are faced with additional specific tasks of teaching linked to language.

Studies of MKT in multilingual contexts outside Africa where the students are English language learners in an English-speaking country are also limited. I will focus on the work of Sorto, Wilson, and White (2018) and Wilson (2016) at Texas State University in the US. They studied specifically MKT for teaching English language learners and developed from Ball et al. (2008). Lessons we get from their studies are that there is a special kind of extra knowledge that teachers of English language learners require and that this knowledge fits within the Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) domains of MKT. They classify this knowledge as knowledge of obstacles, knowledge of resources, and knowledge of strategies. They represent these as in fig. 2 which they call pedagogical content knowledge for teaching mathematics to English language learners (PCK-MELL).

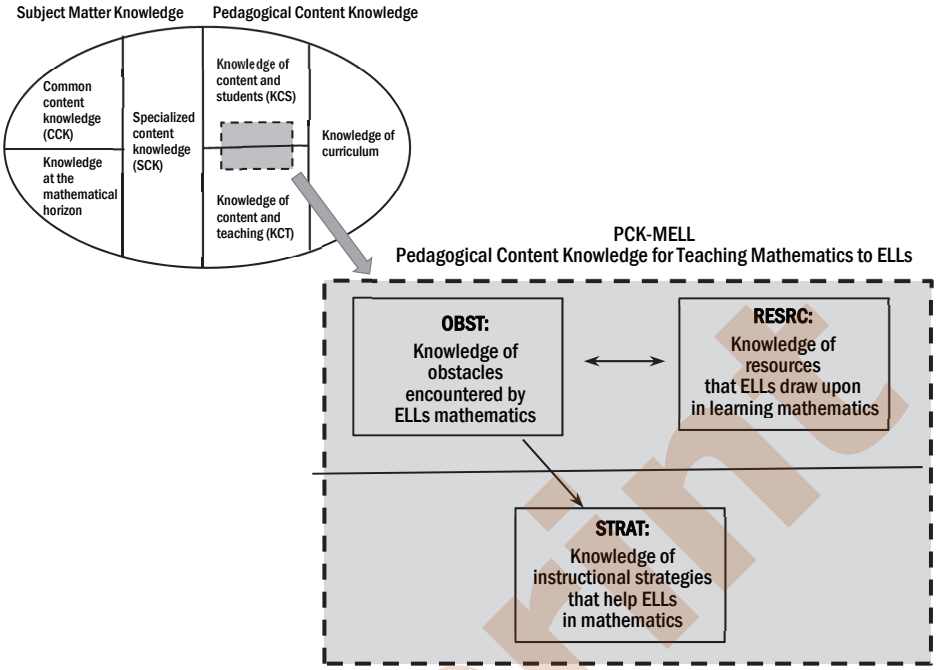

Fig. 2 PCK-MELL (Sorto et al., 2018, p. 222)

Another lesson we learn from these studies is that this knowledge can be measured using carefully designed items, similar to measuring the other knowledge of MKT domains. Sorto et al. (2018) aim was to identify domains of knowledge. My emphasis differs in that I aim to identify the mathematical work of teaching.

4. Lessons from my work in Malawi

My work in Malawi over two decades has been about teaching mathematics in multilingual contexts, mathematical knowledge for teaching, teacher education and professional development of teacher educators and teachers. In my earlier studies I focused on language and mathematics, then later and until now on mathematical knowledge for teaching. Since 2013, my work has mostly been both research and development projects on teacher education and professional development of teachers. I am privileged to work in collaboration with colleagues from University of Stavanger. In this collaboration we have conducted two large projects, the first was from 2014 to 2018 and titled: Improving quality and capacity of mathematics teacher education in Malawi. The second project is titled: Strengthening numeracy in early years of primary education through professional development of teachers, currently in progress, started in 2017 and will end after 2021. I also work with colleagues from Malawi some of whom started as PhD students in the projects.

In Tab. 1 below, I present a summary of selected and relevant studies that I have participated in over the years.

Tab. 1 Relevant studies in Malawi

Year	Study	Researchers
2000–2002	Students understanding of probability vocabulary	Kazima
2010	Code switching in primary mathematics classrooms	Kazima and Pwele
2011	Using bilingual approach in standard 6 mathematics	Kazima, Pwele, and Kasakula,
2014–2015	Language and students' conceptions of logic in undergraduate mathematics	Levis Eneya, Mercy Kazima, Patrick Sawerengera
2014–2015	Exploring mathematical tasks of teaching in Malawi schools	Kazima and Jakobsen
2014–2018	Improving quality and capacity of mathematics teacher education in Malawi	Kazima, Jakobsen, Mosvold, Bjuland, Fauskanger, Eneya, Mwadzaangati, and Mamba
2015–2018	Measuring development of MKT in prospective primary school teachers	Kazima, Jakobsen, and Kasoka
2015–2018	Investigating MKT for teaching equations	Mamba
2015–2018	Exploring MKT for teaching geometric proofs	Mwadzaangati
2017–2021	Strengthening numeracy through professional development of mathematics teachers in Malawi	Kazima, Jakobsen, Fauskanger, Hedgevold, Mosvold, Bjuland, Eneya, Mwadzaangati, Mwale, Gobede, and Longwe
2017–2020	Investigating mediation strategies used by early years mathematics teachers in Malawi	Gobede
2017–2020	Making sense of MKT: insights from primary preservice teachers in Malawi	Jacinto
2017–2020	Exploring how primary teacher education prepares pre-service teachers to teach number concepts and operations	Longwe

4.1. Lessons from the studies on language in Malawi

Findings from these studies in Malawi supported findings reported in literature, and the lessons drawn are similar. For these studies specifically the lessons are:

(1) Students' meanings for mathematical terms can be different from the mathematical meanings, and these are influenced by their home languages (Kazima, 2007; Kazima, Eneya, and Sawerengera, 2015)

(2) Code switching can be used effectively to make mathematics accessible to students (Kazima, Pwele, and Kasakula, 2011)

(3) Bilingual approach where use of home language is planned and proactive can be effective in making mathematics accessible to learners (Kazima, Pwele, and Kasakula, 2011)

There are at least two other researchers that have studied language and mathematics in Malawi: Chitera (2009a, 2009b) and Kaphesi (2002). Chitera (2009a, 2009b) studied teacher education and the discourse practices in preservice mathematics education classrooms. She found that English is used in all lessons and there is no

reference to home languages. The lesson we draw from Chitera's work is that teacher education does not prepare teachers for the teaching in multilingual contexts. Our ongoing project work with mathematics teacher educators in Malawi confirms this. Kaphesi's (2002) work analysed language practices of primary school teachers of mathematics. His findings confirm that code switching is the common practice that teachers use in teaching mathematics in Malawi. The lesson drawn from this is therefore similar to earlier lessons, that code switching between English and home language can be an effective strategy in teaching mathematics to learners that are not fluent in English.

4.2. Lessons from the studies on MKT in Malawi

It has been interesting to use theories developed in the US which is a context different from Malawi. Our use of the MKT framework was informed by many other researchers that used the framework outside the US. For example in South Africa where some of the school contexts are similar to Malawi. General findings from these studies on MKT in Malawi include:

(1) All the tasks of teaching suggested in the US were viewed as relevant by Malawi teachers but at varying levels of importance (Kazima, Jakobsen, and Kasoka, 2016).

(2) All MKT domains were observed in Malawi teaching demands (Mwadzaangati, 2018; Mamba 2018).

(3) Adapted MKT measures were appropriate to use in Malawi context (Jakobsen, Kazima and Kasoka, 2018).

Lessons from these findings are that the general tasks of teaching mathematics are the same in Malawi as elsewhere. However, language always surfaced as an issue, indicating that there are other specific demands on teaching related to language.

5. Conceptualising the Work of Teaching in Multilingual Contexts

Learning from all the discussion so far, it appears that teaching in multilingual contexts makes additional demands on teachers. I will focus on the context of Malawi and draw from the previous studies to conceptualise the mathematical work of teaching in this context. I do this by first considering the lessons drawn from studies on teaching mathematics in multilingual contexts. I take these as professional knowledge of teaching mathematics in multilingual contexts. Professional knowledge because they inform the mathematics education field and profession. From the professional knowledge, I identify the specific knowledge demands on teaching, then further identify the mathematical work of teaching that teachers face. I present this in form of a table as shown below.

As seen in Tab. 2, I propose four categories of mathematical work of teaching that teachers face: (1) identifying obstacles in home language, (2) identifying obstacles in English, (3) identifying resources in home language, and (4) identifying strategies; strategies to address the obstacles and strategies to draw from the resources in home

language. I give some examples below to illustrate each of the proposed mathematical work of teaching

Tab. 2 Professional knowledge, specific knowledge demands and mathematical work of teaching in multilingual contexts

	Professional knowledge of teaching mathematics in multilingual contexts	**Specific knowledge demands on teaching**	**Mathematical work of teaching**
1	Students' meanings for mathematical terms can be different from the mathematical meanings, and these are influenced by their home languages	Knowledge of words in home language that are equivalent or closest equivalent to mathematical terms Knowledge of different meanings of the home language words that can cause difficulty. Knowledge of words in home language that students draw on to make meaning of the mathematical terms Knowledge of how to use the home language words in mathematics	Identifying resources in home language Identifying obstacles in home language Identifying strategies to draw from the resources Identifying strategies to address the obstacles
2	Code switching can be used effectively to make mathematics accessible to students	Knowledge of words in English that students find difficult Knowledge of words in home language that are equivalent or closest equivalent to the English words Knowledge of different meanings of the home language words that can cause difficulty. Knowledge of how to use the home language words in mathematics	Identifying obstacles in English Identifying resources in home language Identifying obstacles in home language Identifying strategies to draw from the resources Identifying strategies to address the obstacles
3	Code switching presents challenges and dilemmas for teachers		
4	Bilingual approach where use of home language is planned and proactive can be effective in making mathematics accessible to learners		
5	Home language should be viewed as a resource rather than a problem	Knowledge of what students can draw on in the home language to make sense of the mathematics being taught Knowledge of how to use these in mathematics	Identifying resources in home language Identifying strategies to draw from the resources

5.1. Identifying obstacles in home language

In drawing from home languages there are at least two types of obstacles (i) equivalent words in home language not as precise as the mathematical words and (ii) non-existence of equivalent words in home language.

5.1.1. Example of equivalent words in home language not as precise as the mathematical words

One example in Chichewa is the concept and operation of multiplication. Multiplication is translated as kuchulukitsa which literary means to increase or to make more. Being able to identifying the obstacle of using kuchulukitsa requires understanding the limitation of describing multiplication as to increase or make more. Specifically it requires understanding that it is true only for positive numbers multiplied by numbers more than 1; and that it is not only multiplication that can result in an increase, other operations on numbers can also result in an increase. For example, adding a positive number or dividing by a positive fraction. The work of teaching is to

identify such obstacles and find ways of addressing them. This is mathematical work and is what teachers face for effective teaching of multiplication in this context.

Another example is the equal sign (=). In Chichewa it is translated as zikhala, which literally means will become. Thus the meaning assigned to the equal sign is that of getting a result after performing an operation. The limitation of zikhala is that it does not convey the equivalence meaning of the equal sign. For instance, showing equivalent fractions, such as $2/3 = 4/6 = 6/9 = 8/12 = \dots$, requires the equivalence meaning of the equal sign.

5.1.2. Example of non-existence of equivalent words in home language

There are some concepts such as similarity which do not exist in Chichewa and are difficult to explain. Similar is translated as kufanana literary meaning look alike, there is no word for proportional and is very difficult to explain proportionality in Chichewa. The work faced by teachers is to identify such obstacles in the home language and find other ways of explaining, such as using many examples of what are and what are not similar in mathematics, as well as the limitation of thinking of similar as only *kufanana.*

5.2. Identifying obstacles in English

There are many obstacles in English for learners that are not fluent in the language. The work for teachers is to observe closely and identify these. For example, sound alike words such as *size*, *side* and *sight* have been found to be used by learners interchangeably (Adler, 2001) and the words *probability*, *disability* and *ability* were thought to mean the same by some learners in a secondary school mathematics lesson on probability (Kazima and Adler, 2006; 53). Kazima and Adler call this "hearing disconnects" and argue that it adds to the description of mathematical knowledge for teaching. I emphasise the argument and add that it is a recurrent task that teachers face in this context. Hearing what students say and being careful in how to pronounce the words, as well as anticipating what students might hear and might mean mathematically are part of the mathematical work of teaching.

5.3. Identifying resources in home language

Resources in home language include (i) the words that teachers chose to use to provide mathematical explanations in the home language; and (ii) mathematical concepts in the home language that can be deliberately sought for use in teaching. The first case of identifying words to use mostly happens during lessons as teachers teach and code switch between English and the home language. Teachers are challenged to think fast on the spot while moving the lesson forward. For instance, looking at the two examples of multiplication and equal sign discussed above, after identifying the obstacles in home language, further work for the teacher is to identify words in home language that can be used to explain the concepts in a way that avoids the obstacles. Identifying words in home language also happens during planning where teachers have time to think and plan carefully the words to use in teaching. This is a major part of the

bilingual approach where the use of home language is done proactively rather than reactively (Setati et al., 2008). The planning for this approach requires a lot of time and mathematical reasoning for teachers to provide mathematically appropriate and accurate versions of written materials for the lessons (Kazima et al., 2011).

For the case of mathematical concepts in the home language, one example is that of the number system in Chichewa. The number system has combination of base 5 and base 10, which can be drawn upon when teaching number bonds, number bases and other number concepts and operations. The counting in words is as in Tab. 3:

Tab. 3 Counting in Chichewa

Numeral	Number words in chichewa	Literal meaning in numerals
1	Chimodzi	1
2	Ziwiri	2
3	Zitatu	3
4	Zinayi	4
5	Zisanu	5
6	Zisanu ndi chimodzi	$5 + 1$
7	Zisanu ndi ziwiri	$5 + 2$
…		
10	Khumi	10
11	Khumi ndi chimodzi	$10 + 1$
…		
19	Khumi ndi zisanu ndi zinayi	$10 + 5 + 4$
20	Makumi awiri	10×2
…		
28	Makumi awiri ndi zisanu ndi zitatu	$(10 \times 2) + 5 + 3$
…		
70	Makumi asanu ndi awiri	$10 \times (5 + 2)$
…		
99	Makumi asanu ndi anayi, ndi zisanu ndi zinayi	$(10 \times (5 + 4)) + 5 + 4$
100	Zana	100
864	Mazana asanu ndi atatu makumi asanu ndi awiri ndi zinayi	$(100 \times (5 + 3)) + (10 \times (5 + 2)) + 4$

The mathematical work for teachers is to identify such mathematical resources and make decisions of what and when to use in teaching.

5.4. Identifying strategies

Identifying strategies is in two parts: identifying strategies to address the obstacles and identifying strategies to draw from the resources in home language. The mathematical work of identifying strategies is linked to all the other three mathematical work discussed above. For example, if a teacher decides to teach division by using some activities of equal sharing, then he or she would need to draw upon the words that the children use when sharing and find ways of using those words in teaching that would convey the mathematical concept of division. Thus as the teacher is identifying strategies to draw from the resources in home language, they are also identifying resources in home language and identifying obstacles in home language as well as identifying obstacles in English and identifying strategies to address the obstacles.

Another example, if a teacher decides to use the strategy of word origins to explain meanings of mathematical terms such as polygon, triangle, or isosceles, then the teacher would need translations of the word origins into the home language. The mathematical work is therefore linked to identifying resources in home language, identifying obstacles in home language and identifying obstacles in English. One can see that the mathematical work of identifying strategies is not independent of the other mathematical work, however, it is important to acknowledge as its own category of mathematical work of teaching because it makes specific knowledge demands on teaching and requires special mathematical problem solving by the teachers to decide on the best strategy to use for effective teaching of the specific mathematics content in the multilingual context.

Comparing these identified mathematical work of teaching to the PCK-MELL knowledge domains (Sorto et al., 2018), it can be noted that the categories of mathematical work of teaching and the categories of knowledge domains are similar: they both have categories of obstacles, resources and strategy. I take this similarity as evidence of support for the findings of Sorto et al (2018), although my focus is not knowledge domains. While there is this similarity in categories, there are some differences in the details, which appears to suggest that there is mathematical work of teaching that is common among multilingual contexts, as well as specific mathematical work of teaching for specific multilingual contexts, such as Malawi.

6. Conclusion

In this paper I have conceptualised mathematical work of teaching in multilingual contexts. I have done this by drawing from the professional knowledge of teaching in multilingual contexts that we learn from studies of teaching mathematics in these contexts. Considering the professional knowledge, I first identify specific knowledge demands on teaching then from these further identify the mathematical work of teaching entailed. I identify four categories: identifying obstacles in home language, identifying obstacles in English, identifying resources in home language, and identifying strategies. My approach is different from earlier studies on mathematical work of teaching (e.g. Ball et al. 2008; Adler, 2010) where they start from classroom practice and analyse the work faced by teachers as they teach mathematics. Some might argue that my approach, that is almost the other way round, could miss some details of what teachers are faced with when teaching mathematics. While I appreciate the limitation my approach might have, I would like to highlight that the professional knowledge I have presented is drawn from findings of classroom studies and therefore captured what goes on in practice in the multilingual context.

This identification of mathematical work of teaching in multilingual contexts is important because it informs us that there is specificity to teaching multilingual contexts. Looking at the specific case of Malawi, it appears that the specific nature of the mathematical work of teaching depends on the nature of the multilingualism. The Malawi multilingual context is similar to some that have national languages or

common home languages between the students and the teachers. The context is different from others that do not have such common languages. Nevertheless, the conceptualisation of the mathematical work of teaching that I suggest can also inform those contexts.

Acknowledgement

I acknowledge Reidar Mosvold with great appreciation for insightful comments during preparation of this paper.

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